A Step Length Estimation Model for Position Tracking

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Abstract—Inertial Measurement Unit (IMU) is one option for the positioning system. Due to its independence and invulnerability, the IMU-based approaches could serve as an effective complementarity for the positioning systems applying communication networks, when the infrastructures are insufficient or unreliable. For pedestrians the Step and Heading System (SHS) is a practicable solution. With the length and heading of each step measured by the built-in inertial sensors in users portable device, the current location would be updated. A novel mathematical model for step length estimation is developed in this paper. In this model the relation among the step length, frequency and the variance of accelerations is revealed. Comparing with former models, not only the accuracy of step length estimation is improved substantially, but also the stabilization as well as robustness of the whole positioning system can be enhanced.

Index Terms—Inertial Measurement Unit (IMU), Step and Heading System (SHS), Pedestrian Dead Reckoning (PDR), Position Tracking

I. INTRODUCTION

The development of informative life and the popularization of smartphone have brought the field of indoor positioning great opportunity. The mainstream schemes are more likely to rely on communication networks. For example, as the scheme based on WLAN, the radio strength fingerprint in a certain building is analysed statistically [1][2]. But if there is no available wireless access point in the target building, or the Wi-Fi cover is not complete, this scheme will be limited. The same shortage is also found in the schemes based on RFID or Bluetooth etc. Therefore a complementary strategy has to be developed, which does not lie on external signals. When the communication signal is temporarily unavailable or unreliable, this strategy can be adopted to independently maintain the function of whole positioning system. Inertial Measurement Units (IMU) [3][4] is an ideal alternative. Due to their interference immunity, the built-in inertial sensors within smartphones (or others portable devices) have received increasing attentions. Because the information interaction with external signals is not necessary, this self-contained strategy has a promising prospect in all sorts of Location Based Service (LBS), especially in some extreme situations, such as the rescue mission after earthquake or conflagration in a damaged architecture, or archeological and scientific expedition in a cave or underground where the navigation signals from satellite or cellular network are usually blocked.

For pedestrian a practicable method is to develop a Step and Heading System (SHS), estimating the length of each step by the accelerations recorded when user walking, and according to the heading direction during each step, the real time position is tracked [5][6]. On the basis of our previous works [7][8], before step detection, a variety of calibrations as well as filters are implemented to make the noises and deviations in raw data to the minimum. After that the feature vectors from acceleration sequence are extracted for step length estimation and the gyroscope and magnetometer data are fused for azimuth. Finally Particle Filter is employed to constrain the moving trajectory to the physical surroundings.

Obviously the performance of a positioning system based on Dead Reckoning depends essentially on the precision of step length estimation. Even tiny deviation in every single step could cause troublesome cumulative error in the final tracking result. There are 9 kinds of estimation models available (listed in Section II). All of these current models, complicated or concise, show limitations in the estimation accuracy or universality. The usability of some models must be under certain conditions. For that reason, a novel mathematical model is developed in Section III. In this model the step length is derived from both the step frequency and the variance of vertical accelerations. As a result, more satisfactory precision for step length estimation can be achieved.

II. RELATED WORKS

According to the previous literatures, there are 9 mathematical models available for step length estimation [9].

Static models:

1) Constant step length: Some applications employ this method. The users have to measure their step length by theirselves (walk with a certain amount of steps and measure the distance), or use the recommended value by default.

2) Determined by height: The step length for male is 0.415 times his height while for female is 0.413 her height [10].

Dynamic models:

Of course the 2 static models above are very rough and only capable of some imprecise applications.

3) Weinberg Model: [11]

$$step_length = k \cdot \sqrt[4]{a_{max} - a_{min}} , \qquad (1)$$

where a_{max} and a_{min} denote the peak and trough value of the vertical accelerations in that step; k is the user specified parameter.

4) Kim Model: [12]

$$step_length = k \cdot \sqrt[3]{\frac{\sum_{i=1}^{N} |a_i|}{N}}.$$
 (2)

There are N sampling points in the step and a_i is the vertical acceleration of the *i*th point; k is the user parameter. In this model, the magnitude of acceleration (the expression in cubic radical sign) is drawn on.

5) Scarlett Model: [13]

$$step_length = k \cdot \frac{\frac{\sum_{i=1}^{N} |a_i|}{N} - a_{min}}{a_{max} - a_{min}}.$$
 (3)

The explanations for parameters are the same as above. The peak and trough values as well as the magnitude are utilized. *6) Xu Model:* [14]

$$step_length = k \cdot [(a_{max} - a_{min}) + \sqrt[4]{a_{max} - a_{min}}].$$
(4)

Only the difference between the peak and trough value is made use of.

7) Frequency related models: Some researchers also maintain that there is a linear relation between step length and step frequency [4][15][16]. Although the concrete mathematical expressions are various, the basic models are like this:

$$step_length = af + b,$$
 (5)

where f denotes the step frequency, while a, b are the user parameters. Furthermore, more precise nonlinear model is presented as well [17]:

$$k_d = 1.5k_f^2 - 1.8475k_f + 1.3468, (6)$$

where $k_d = d/d_n$ and $k_f = f/f_n$, d_n and f_n are the step length and frequency when pedestrians walk with their most normal gait (or their average values).

After experiments, there is a fact that, all of linear and nonlinear models above are based on an assumption, or a premise: all of volunteers are required to walk with their normal or comfortable gaits. It can be therefore rationally explained that, why the larger frequency the pedestrian walks with, the longer step length s/he makes. However, the utilization of these models is limited. They are only suitable for the most normal gaits, small steps with high frequency and large paces but low frequency are naturally never referred to.

8) Shin Model: [18]

$$step_length = af + bv + c.$$
 (7)

In this model, not only the step frequency but also the variance during that step is involved. So it is more precise than the frequency singly related models listed before. 9) Bylemans Model: [19]

$$step_length = \sqrt[2.7]{\frac{\sum_{i=1}^{N} |a_i|}{N} \cdot \sqrt{\frac{k}{\sqrt{\Delta t \cdot (a_{max} - a_{min})}}}} \cdot 0.1,$$
(8)

where Δt is the duration of the step (in *ms*), the meanings of the rest of the parameters are similar to those of the models before. In Eq. (8), the magnitude of accelerations, the frequency and the difference between peak and trough are involved. This model is up to now the most meticulous one and was also employed in our previous schemes [7][8].

III. STEP LENGTH ESTIMATION MODEL

A. Deduction for the Model

Although some of the current models already show their conveniences, most of them are only suitable for the most common gait for human (step length: 0.65 to 0.75 meters, frequency: 90 to 120 steps per minute). When it comes to more varied and abundant gaits, the effect is less than satisfactory. For more precision of step length estimation as well as positioning, a novel mathematical model is investigated in this paper. This new model should be general for all sorts of gaits, because the pedestrian activities may also include wander and roam. For the application scenario of museum or exhibition hall, a lower walking speed is more preferred, while a higher speed is geared towards the walking race.

In order to find out the objective relations among the step length, the magnitude of accelerations, the peak value, the trough value, the variance and the frequency of that step, a multitude of experiments are implemented.

The experiments are based on the accelerometer in smartphone. The sampling rate is set to 50 Hz. All the acceleration related data are measured in G, which denotes 1 unit of gravity acceleration (ca. 9.8 m/s^2). The step length is set from 0.4 to 0.9 m, every 0.05 m a group, and 11 groups in all; while the frequency varies from 60 to 180 spm, every 5 spm a group, and 25 groups in all, where spm stands for steps per minute. For every step length group, and every frequency group: the magnitude, average peak value, average trough value, and the variance for acceleration data in both anterior-posterior direction (y-axis), vertical direction (z-axis) and their module (m) are recorded (12 items in all). The definition of magnitude is the same as that in Eq. (2), (3) and (8). Each combine group contains a certain step length and a certain frequency, so there are 275 combine groups in all. In order to reduce random error, all of these experiments are implemented by just one volunteer, and every combine group with the same gait is carried out at least 10 times. All these experiments were performed in the corridor of our institute (as Fig. 1). A treadmill is not adopted due to the potential influence of conveyer belt in y-accelerations. The walking distance for 1 time is between 20 and 30 m (20 to 50 steps). So the whole walking distance for this volunteer is more than 50 km. The first stage for experiments lasted for 3 weeks (The following stages are for running and putting smartphone in pocket).



Fig. 1. To discover the step length model, a large number of experiments are implemented by volunteer. The step length and frequency are calibrated by measuring tape and metronome. The acceleration data are collected by smartphone.



Fig. 2. The step lengths are preselected

As shown in Fig. 1, to adjust the volunteer's steps to a preselected frequency, a metronome app for music is utilized. To make the volunteer's steps coincide with a certain length, a measuring tape is laid on the floor and all foot striking points are marked with color taps (Fig. 2). After 3 weeks walking experiments, a considerable quantity of data are acquired. An assumption has to be accepted that the changing in volunteers weight could not influence the collected data greatly.

To demonstrate the relations between the 8 items and frequency, the data when the step length = 0.7 m are shown in Fig. 3 as an example. From top to bottom they are data in *y*-axis (anterior-posterior direction), *z*-axis (vertical direction) and their module respectively. From left to right they are the magnitude, the difference between peak and trough, and the variance successively. Because there is too much confusion in the peak-trough difference of the module values, the figures for this item are not displayed. Instead of peak and trough value separately, the differences between them are used.

From Fig. 3, it can be found that, the relations in terms of y-data and m-data are not clear. In view of that, only z-data related items would be involved in our model.

The aim of the model is to discover a relation that the step length can be expressed by the measured data. Fig. 4 shows in different frequencies, the variation trend of the 3 items in terms of step length. Due to the instability in y-data, only the items about z-data are referred to. Here each



Fig. 5. The fitting curves of the step length to the 3 items (the magnitude, the difference of peak and trough, and the variance) are shown according to different frequency groups.

frequency group is defined as frequency domain rather than a certain frequency value. Because during the experiences it is too difficult for volunteer to adjust to a definite frequency always accurately, there are some deviations in the frequencies measured afterward. From Fig. 4, it is evident that there are quadratic relations between these 3 items and step length. By defining just one of these 3 relations, the step length model can be acquired. For each of 11 frequency groups and each of 3 items, there would be a fitting curve. The 33 fitting curves are therefore compared in Fig. 5.

The item *variance* is chosen as the independent variable for the final model. Because from Fig. 5, the curves in the bottom figure vary most sharply. Among these 3 items, the *variance* shows the most remarkable relation with the step length. The model is expected be in the form that step length is a function of *variance* and *frequency*.

According to Fig. 5, it is assumed that the relation between variance (v) and step length (s) is quadratic:

$$v = as^2 + bs + c, (9)$$

where a, b, and c are coefficients to be determined. These coefficients are also functions in terms of *frequency*. In order to determine these coefficients, the relations between *frequency* and *variance* under certain step length are required. These relations can be obtained from the fitting curves. As Fig. 6,



Fig. 3. With 0.7 m as step length, the 8 items vary with different frequencies



Fig. 4. The relation between the step length and the 3 items (the magnitude, the difference of peak and trough, and the variance) are shown according to different frequency groups.



Fig. 6. When step length is 0.7 m, the relation between *frequency* and *variance* is shown. Two quadratic functions can be fitted.

 TABLE I

 FREQUENCY-VARIANCE RELATIONS AT DIFFERENT STEP LENGTHS

Step length	Frequency-Variance relations	
0.4	$v = 1.61 \times 10^{-8} f^2 + 0.0000203 f - 0.00076$	8 (f < 140)
	$v = 0.00000214f^2 - 0.000776f + 0.0722$	$(f \ge 140)$
0.5	$v = 1.52 \times 10^{-8} f^2 + 0.0000344 f - 0.00141$	(f < 140)
	$v = 0.00000483f^2 - 0.000924f + 0.0779$	$(f \ge 140)$
0.6	$v = 0.00000221f^2 + 0.000293f - 0.0113$	(f < 140)
	$v = 3.075 \times 10^{-7} f^2 - 0.000155 f + 0.0256$	$(f \ge 140)$
0.7	$v = 0.00000650f^2 - 0.000951f + 0.0386$	(f < 140)
0.7	$v = 0.00000750f^2 - 0.00285f + 0.283$	$(f \ge 140)$
0.8	$v = 0.00000826f^2 - 0.000888f + 0.0287$	(f < 140)
0.8	v = -0.000719f + 0.147	$(f \ge 140)$
0.9	$v = 0.0000128f^2 - 0.00131f + 0.0407$	(f < 140)
0.9	v = -0.00135f + 0.263	$(f \ge 140)$

this example is f - v figure when step length is 0.7 m. This figure is already shown as a part of Fig. 3.

When step length is 0.7 m, the relation between *frequency* and *variance* can be expressed by a piecewise function:

$$\begin{cases} v = 0.00000650f^2 - 0.000951f + 0.0386 \ (f < 140) \\ v = 0.00000750f^2 - 0.00285f + 0.283 \ (f \ge 140) \end{cases}$$
(10)

Along with the increasing frequency, the motion in vertical direction would be firstly more and more intense, and then calmer and calmer. Similarly, the relations between frequency and variance at other step lengths (part of) are listed in Table I.

With the 6 pairs of equations in Table I, the coefficients a, b, c can be calculated by the least square method:

$$\begin{bmatrix} c\\b\\a \end{bmatrix} = \begin{bmatrix} 6 & \sum_{i=1}^{6} s_i & \sum_{i=1}^{6} s_i^2\\ \sum_{i=1}^{6} s_i & \sum_{i=1}^{6} s_i^2 & \sum_{i=1}^{6} s_i^3\\ \sum_{i=1}^{6} s_i^2 & \sum_{i=1}^{6} s_i^3 & \sum_{i=1}^{6} s_i^4 \end{bmatrix}^{-1} \begin{bmatrix} \sum_{i=1}^{6} v_i \\ \sum_{i=1}^{6} s_i v_i \\ \sum_{i=1}^{6} s_i^2 v_i \\ \sum_{i=1}^{6} s_i^2 v_i \end{bmatrix}$$
(11)

where s_i are [0.4 0.5 0.6 0.7 0.8 0.9]; v_i are the equations in Table I. Therefore, when f < 140 spm:

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0.0000545f^2 - 0.00501f + 0.15495 \\ -0.0000461f^2 + 0.00404f - 0.130 \\ 0.0000102f^2 - 0.000913f + 0.0336 \end{bmatrix}; (12)$$

when $f \ge 140$ spm:

s

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0.000178f^2 - 0.0613f + 5.381 \\ -0.000177f^2 + 0.0607f - 5.272 \\ 0.0000423f^2 - 0.0145f + 1.248 \end{bmatrix}.$$
 (13)

From Eq. (9),

$$r = k \cdot \frac{-b + \sqrt{b^2 - 4a(c - v)}}{2a}.$$
 (14)

The Eq. (12), (13) together with (14) makes up the main ingredients of the mathematical model for step length estimation. The step length is finally a function of *frequency* and *variance*.

However, all of sampling data source from 1 volunteer. So theoretically the equations above are only suitable for that 1 volunteer. When a pedestrian walks calmly, with the same step length a comparative lower variance would be measured, or when the feet strike ground harshly a higher variance resulted. Even the gesture or position holding smartphone can influence the variance greatly. In order to generalize the model, the individual parameter k is added to the model.

One or more test walking can be used for parameters calibration. The distance could be estimated with the initial k=1, and compared with the real distance entered by user. Eq. (15) is used to set the user specified parameter.

$$k = d_{real}/d_{estimated},\tag{15}$$

where $d_{estimated}$ and d_{real} denote the distances estimated with the initial parameter and the real distance during test walking. Adding with Eq. (15), the model is completed.

B. Evaluation for the Model

To test the precision of our model for step length estimation, 10 different volunteers (male and female, height from 1.60 m to 1.80 m, weight from 60 kg to 75 kg) are required to walk a distance at least 30 meters with smartphone held in different gestures (except swinging or changing between gestures). They are encouraged to walk with their normal and abnormal step lengths as well as frequencies.

After all of test experiments, the estimation precisions of all models are listed in Table II. The precision is indicated by 2 indexes: Root Mean Squared Error (RMSE) and Average Deviation Rate.

The deviation rate is calculated by

$$deviation_rate = |\frac{s_{estimated} - s_{real}}{s_{real}}| \times 100\%$$
(16)

where $s_{estimated}$ denotes the step length estimated by different models, while s_{real} is the real observed value.

From the results listed in Table II, Weinberg and Xu Models are both based on the difference between peak and trough. Their precisions are on a similar level. Xu Model has even precisions toward different step lengths but Weinberg Model performs better in 0.7 to 0.9 m length domain in which pedestrians usually walk. Frequency related models show less precision in the experiments. The reason has analysed before, they are only suitable for the most common gaits (step length

TABLE II								
THE RMSES AND AVERAGE DEVIATION RATES OF ALL AVAILABLE MODELS								

Step Length Domains (m)		0.40-0.49	0.50-0.59	0.60-0.69	0.70-0.79	0.80-0.89	0.90-0.99	All
Weinberg	RMSE	0.16951	0.10327	0.07335	0.06191	0.07402	0.11029	0.10089
Model	Average Deviation Rate	41.61%	18.55%	10.14%	7.58%	8.26%	9.84%	15.79%
Kim	RMSE	0.17543	0.08457	0.08985	0.09550	0.10316	0.10242	0.10655
Model	Average Deviation Rate	43.37%	14.14%	12.85%	12.01%	12.14%	8.66%	16.34%
Scarlett	RMSE	0.34873	0.19269	0.10021	0.01141	0.09738	0.19708	0.17335
Model	Average Deviation Rate	86.95%	38.34%	16.36%	1.37%	12.25%	21.98%	28.11%
Xu	RMSE	0.10489	0.07248	0.09493	0.12298	0.17271	0.15061	0.11661
Model	Average Deviation Rate	24.36%	12.03%	12.67%	14.65%	18.85%	13.15%	15.24%
Frequency	RMSE	0.37316	0.32566	0.26572	0.18114	0.13944	0.24883	0.26419
Model	Average Deviation Rate	87.40%	57.01%	35.75%	20.98%	15.58%	26.44%	40.71%
Shin	RMSE	0.33404	0.28898	0.23151	0.16262	0.13738	0.21270	0.23997
Model	Average Deviation Rate	76.99%	48.62%	30.56%	19.58%	15.03%	22.36%	35.99%
Bylemans	RMSE	0.17029	0.08246	0.08512	0.09099	0.10198	0.15163	0.10835
Model	Average Deviation Rate	42.57%	13.69%	12.73%	11.47%	11.27%	14.17%	16.16%
Our	RMSE	0.13880	0.00685	0.05675	0.05212	0.08716	0.05060	0.07681
Model	Average Deviation Rate	33.17%	13.68%	7.24%	5.88%	8.78%	4.73%	10.90%

0.65-0.75 m, frequency 90-120 spm), but here abnormal gaits are involved. Shin Model also refers to the variance, so its performance is better than only frequency based models. Our model is up to now the most accurate model in all. The average deviation rate for each step is 10.90%. This is only average precision. In terms of more general step length domains, the deviations are even lower.

IV. CONCLUSION

A novel mathematical model for step length estimation is described in this paper. From the test experiments, the average deviation rate is only 10.90%, which shows more superb performance than other models in the same condition. Although it is complicated and consists of 4 equations (Eq. 12, 13, 14, 15), its computation cost is feasible for smartphone. Besides, there is another advantage in our model. Because the expressions refer to the variance of vertical accelerations rather than the differences between peak and trough, our model is able to promote the stabilization of the whole positioning system. Once any single peak or trough is misrecognized, it could lead to a series of disorders in peak-trough pairs, which would cause meaningless result in peak-trough based models. But the variance can be safely measured and the influence of possible error in frequency is also comparatively limited.

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